Electricity term structure modelling

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Motivation

- Financial electricity forward contracts are settled against the average spot price - electricity swaps
- How can we model the term structure of electricity?
  - Forward curve versus swap model
  - Implementation and estimation
  - Application in mind: European options written on electricity swaps

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Outline

Nordic power exchange: NordPool

- NordPool area: Norway, Sweden, Finland, Denmark
- The day-ahead market:
  - Day-ahead: Price and volume for each of the 24 hours the next day
  - Equilibrium price ignoring congestion: *system price*
  - When congested: Different equilibrium price area
  - Physical players only
- The financial market:
  - Forward/futures and European options
  - Open for all players with a margin account

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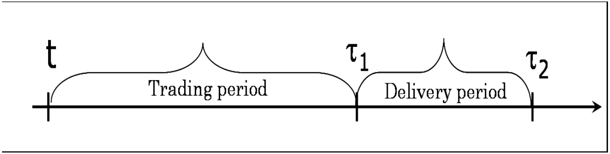
Forwards with settlement period = swaps



- Financial forwards/futures
- Also called swaps: Fixed for floating price
  - Delivery periods: day, week, month, quarter, year
  - European options on swaps forwards matures prior to delivery period
- Swaps settles against *system price*

The Heath-Jarrow-Morton (HJM) approach

- Use ideas from interest rate theory to model electricity markets
- Forward price dynamics instead of spot
- Heath-Jarrow-Morton 1992:
  - Model the complete term structure dynamics of interest rates directly under the risk-neutral probability
  - Analogue in electricity: Model the term structure dynamics of forward/futures prices in stead of spot

Relationships: Spot - forward - swap

- $S(t)$ is the spot price and $f(t,u)$ is the price of a forward with fixed-delivery time at u,

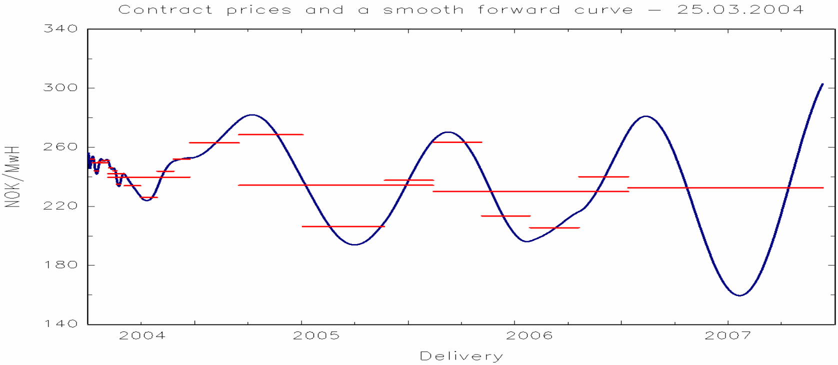
$$f(t,u) = \mathbb{E}_Q\left[S(u) \,|\, \mathcal{F}_t\right]$$

- No-arbitrage relationship between fixed delivery forwards and a swap that settles at τ_2

$$F(t,\tau_1,\tau_2) = \frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} f(t,u)\,du$$

The forward curve must be computed from swaps

- Modelling in 2 steps:
  1. Compute $f(t, u)$ consistent with $F(t, \tau_1, \tau_2)$'s
  2. Come up with a model for $df(t, u)$



The maximum smoothness approach

- Adams and van Deventer (1994) suggested a spline approach ("maximum smoothness") to compute the interest rate forward curves
- Benth et al. (2005) modifies to electricity swaps
- Decompose the forward price into

$$f(u) = s(u) + \varepsilon(u); u \in [u_0, u_n]$$

- Interpretation: $s(u)$ captures seasonality and $\varepsilon(u)$ captures deviation from the seasonality
- The function $\varepsilon(u)$ is 4th order polynomial

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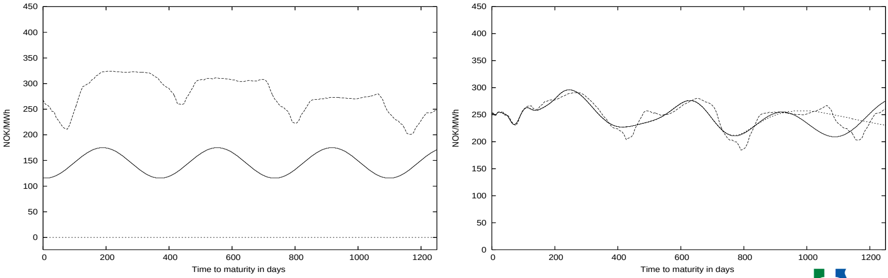
A forward curve comes in many shapes...

- Market data from Nord Pool, May 4. 2005



Multi-factor GBM - model set up

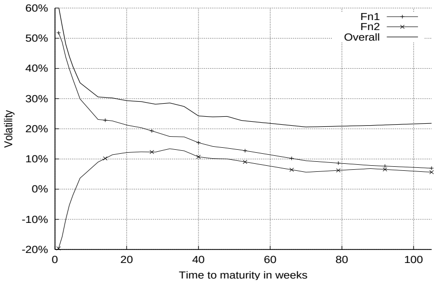
- Koekebakker and Ollmar (2005)
- Assume a market with a continuum of forwards
- Suppose a multi-factor geometric Brownian motion forward dynamics

$$\frac{df(t,u)}{f(t,u)} = \sum_{i=1}^{K} \sigma_i(t,u)dW_i(t)$$

- What does these volatility functions look like?

Multi-factor GBM - Empirical results I

- Compute daily forward curves 1995-2001
- Use Principal Component Analysis (PCA) to decompose covariance matrix of returns
- Compute volatility functions from eigenvectors



Multi-factor GBM - Empirical results II

- Further results:
  1. $K = 2$ factors captures 75% of return variation
  2. Factor 1 shifting, factor 2 tilting factor
  3. $K = 10$ factors captures 95% of return variation
  4. Low correlation between long and short end of the curve
  5. Each point on the curve has both common and idiosyncratic risk?

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Implied swap dynamics

- Assume a market with a continuum of forwards and a finite number of non-overlapping swaps.
- Suppose a geometric Brownian motion forward dynamics

$$df(t,u) = f(t,u)\sigma(t,u)\,dW(t)$$

- Implied swap price dynamics, $t < \tau_1$

$$dF(t,\tau_1,\tau_2) = \Big\{\sigma(t,\tau_2)F(t,\tau_1,\tau_2) - \int_{\tau_1}^{\tau_2} \frac{u-\tau_1}{\tau_2-\tau_1}\partial_2\sigma(t,u)F(t,\tau_1,u)\Big\}\,dW(t)$$

- Not a tractable dynamics except for...

A special case

- In the classical Black model with constant volatility

$$df(t,u) = \sigma f(t,u)\, dW(t)$$

- Implied swap price dynamics

$$dF(t,\tau_1,\tau_2) = \sigma dF(t,\tau_1,\tau_2)\, dW(t)$$

- Log-normal swap dynamics only for maturity independent forward dynamics

- Drawbacks with the fixed-delivery HJM-approach:
  1. Model of non-existing forwards
  2. Estimation uses data which must be transformed (smoothed)
  3. Implied electricity forward dynamics is very involved, even for a GBM-model

HJM swap approach

- Idea in Benth and Koekebakker (2005): Model swap dynamics directly

Problem: Too many overlapping swaps

- No-arbitrage condition: Overlapping forwards must satisfy

$$F(t, \tau_1, \tau_N) = \sum_{i=1}^{N-1} \frac{\tau_{i+1} - \tau_i}{\tau_N - \tau_1} F(t, \tau_i, \tau_{i+1})$$

- If market trades in swaps with *all* possible delivery periods

$$F(t, \tau_1, \tau_N) = \frac{1}{\tau_N - \tau_1} \int_{\tau_1}^{\tau_N} F(t, u, u)\, du$$

- GBM-model with maturity dependent volatility does not satisfy this condition

Solution: Model only non-overlapping swaps

- Hard to state models which are realistic, easy to estimate and satisfy the no-arbitrage condition
- A practical approach: Model only the existing swaps in the market
    1. Single out the “smallest” swaps (the building blocks)
    2. Assume a dynamics for these
    3. Forwards with larger delivery period are priced by the no-arbitrage relation

Option pricing in log-normal swap market model

- Let swap dynamics for non-overlapping swaps $t \leq \tau_1$:

$$dF(t,\tau_1,\tau_2) = \Sigma(t,\tau_1,\tau_2)F(t,\tau_1,\tau_2)\,dB(t)$$

- Question: Price at time t of a European swaption with maturity $T \leq \tau_1$?
- Answer: Replace $\sigma^2(T-t)$ with $\int_t^T \Sigma^2(s,\tau_1,\tau_2)\,ds$ in the Black formula

Volatility term structure - model specification

- Let swap dynamics for non-overlapping swaps for $t \leq \tau_1$:

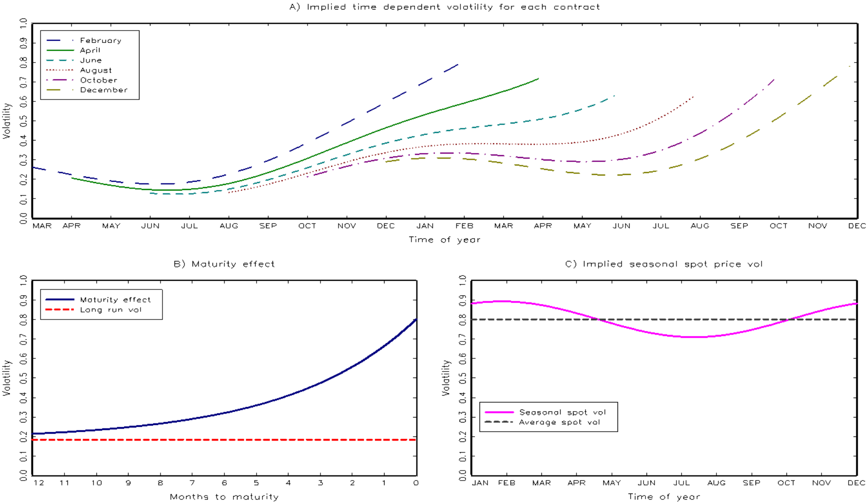
$$dF(t,\tau_1,\tau_2) = \Sigma(t,\tau_1,\tau_2)F(t,\tau_1,\tau_2)\,dB(t)$$

- Fitted to data from NordPool
  - Extracting only non-overlapping forwards
  - Using more than 50.000 price data (1995-2004)
- Assumed constant market price of risk and volatility with separated seasonality and maturity effect

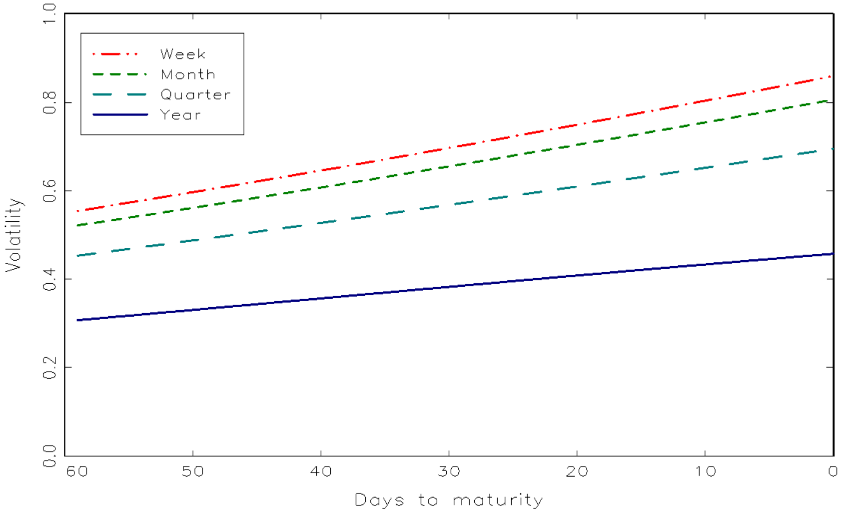
$$\Sigma(t,\tau_1,\tau_2) = \frac{\sigma}{a(\tau_2-\tau_1)}\left\{e^{-a(\tau_1-t)} - e^{-a(\tau_2-t)}\right\} + s(t)$$

- $s(t)$ truncated Fourier series

Volatility term structure - monthly swaps



Volatility term structure - delivery periods



Concluding remarks

- HJM forward curve model
  - Stylized facts: Maturity, delivery period, seasonality, low correlation, idiosyncratic risk
  - Complicated dynamics for market traded swaps
  - Closed form option pricing for constant volatility only
- HJM non-overlapping swap model
  - Closed form option pricing
  - Estimation of one-factor model on historical prices
- Outlook
  - Multi-factor swap models

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