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# Smile trading

Topic Paper Equity Derivatives Technical Study – Europe

## We shall never know all the good that a simple smile can do[1]

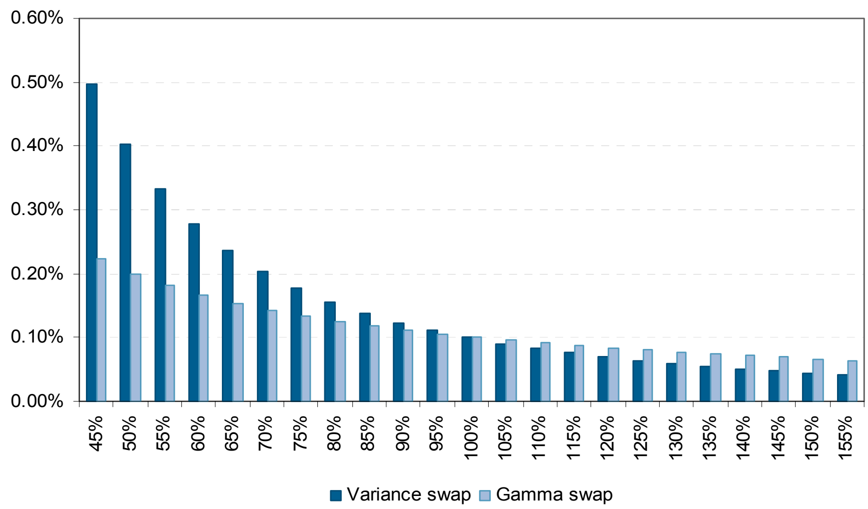
- In this report, we investigate how to make profit from a potential cheap or dear 90-110% smile. While options strategies such as collar or call ratio are sensitive to the smile but not necessarily in a linear way, the difference in strike prices between variance and gamma swaps is linearly related to a change in the smile. We therefore look how to trade the smile, first in theory by showing how variance and gamma swaps are impacted by the smile.

- Since the gamma swap has a slightly positive delta, we then compare the costs and benefits of delta-hedging the position. Given the fact that suggested delta-hedging strategies do not provide a clear delta-neutral position, we argue that the cost of delta-hedging could overcome its benefit.

- Finally, we suggest a simple trading strategy that consists in the following approach: if the 90-110% smile is 1.5-standard-deviation above (below) its historical average, we recommend selling (buying) a variance swap and buying (selling) a gamma swap.

- We have backtested the strategy on the €-Stoxx 50, the FTSE 100, the CAC 40 and the DAX 30 from January 2001 to January 2006. After transaction costs, the strategy would have generated 1.21% on average in variance terms for the €-Stoxx 50 index. Depending on the index considered, the strategy would have performed in 75 to 100% of the cases.

**Variance swaps and gamma swaps' replication using vanilla options**




Source – BNP Paribas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[1] Mother Teresa's quote

# The robbed that smiles, steals something from the thief

William Shakespeare, Othello

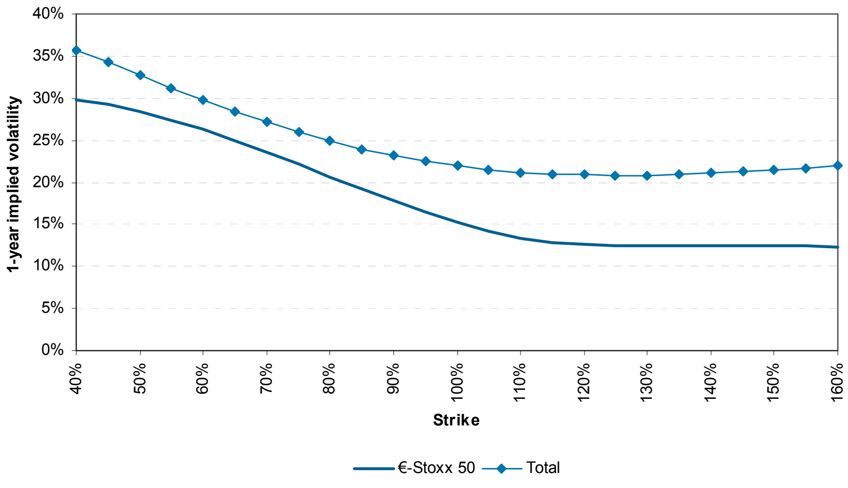
## In search for hidden assets

There has been recently a search for investing into “hidden assets”. Hidden assets include any asset in which it is not possible to invest directly through listed and standardised products and is comprised among others of volatility, dividend, correlation and indeed the volatility smile.

The smile refers to the fact that implied volatility in practice varies across strikes. As the following graph shows, the implied volatility curve usually takes two shapes:

- A smile, when both OTM and ITM volatilities are higher than ATM volatility. This is often the case for stocks.
- A smirk, or skew, when the implied volatility is a decreasing function of the strike price. This pattern is most often observed for indices.

**1-year implied volatility curve for Total and €-Stoxx 50 index as of 12 December 2005**




Source – BNP Paribas

This is indeed one of the major flaws of the Black & Scholes model which assumes a constant implied volatility curve. While the enigmatic smile of Mona Lisa still fails to be deciphered 500 years after it was painted by Leonardo Da Vinci, several theoretical and practical explanations have been put forward for the volatility smile:

- Leptokurtosis: The Black & Scholes model assumes that stock returns follow a normal (or Gaussian) distribution. This implies that returns are evenly and symmetrically distributed on both sides of their mean. In practice, returns are often found to be leptokurtic, i.e. there is a higher probability of a large downward movement than a large positive upward one. Theoretical models using stochastic volatility or distribution with jumps can create this empirical pattern.

- Heterogeneous beliefs: by taking into account heterogeneous beliefs with regard to dividend growth among investors, Buraschi and Jiltskov show that it can explain the smile.

- Correlation risk: in the presence of correlation risk, index's implied volatility yields a smile even if stocks have a flat implied volatility curve.

- Supply and demand: Insurance companies, through the use of OTM put or structured products, are often willing to buy downside protection, which leads to a higher OTM volatility.

## How to make you smile – in theory

Trading variance swaps vs. gamma swaps can help you capture the volatility smile as a camera can do with a smile on your face. A variance swap is a contract, the payoff of which is defined by:

$$Pay-off_{Varianceswap} = (Variance - K_{Var}) * N$$

and where the variance is defined by:

$Variance = \frac{252}{T}\sum_{i=1}^{T}\left(\ln\left(S_i/S_{i-1}\right)\right)^2$ , $K_{Var}$ is the strike and N the notional amount

A variance swap therefore provides a simple and cost-efficient way to get a pure exposure to variance.

The gamma swap's payoff is given by:

$$Pay-off_{Gamma\,swap} = (Gamma - K_{Gamma}) * N$$

Where $Gamma = \frac{252}{T}\sum_{i=1}^{T}\left[\left(\ln\left(S_i/S_{i-1}\right)\right)^2 \frac{S_i}{S_0}\right]$

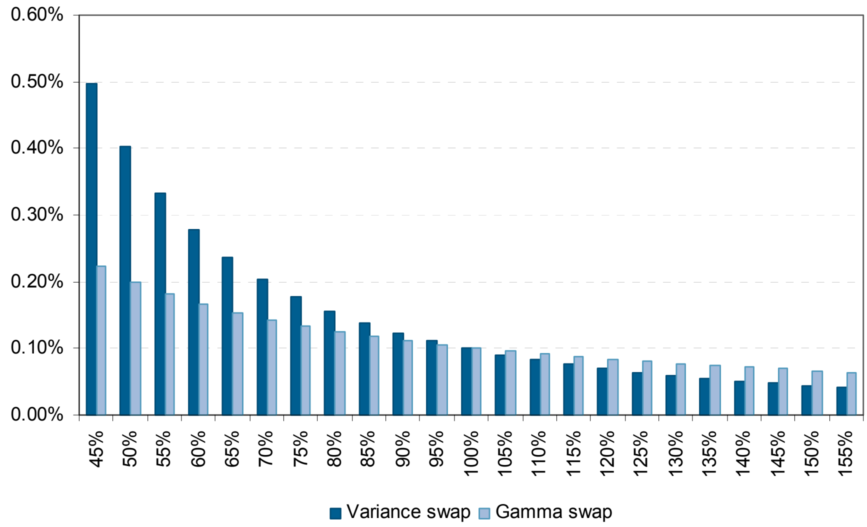
$K_{Gamma}$ is the strike and N the notional amount. Gamma swaps are thus equivalent to variance swaps whose nominal is proportional to the level of the underlying asset.

By virtue of their payoff and insofar as squared returns $\left(\ln\left(S_i/S_{i-1}\right)\right)^2$ are weighted by the performance of the stock $S_i/S_0$ , gamma swaps underweight big downward index move relative to variance swaps. This means that if the distribution of stock returns is skewed to the left, gamma swaps minimise the effect of a crash, thereby making it easier for the trader to hedge. In this case, hedging does not require additional caps, unlike variance swaps which need to be capped when the underlying asset is a single stock.

One can further show that variance swaps can be replicated by a static portfolio of options with a continuum of strike prices and where the each option is weighted by *$1/K^2$*. Gamma swaps can be replicated the same way and as its nominal is proportional to the level of the underlying asset, the gamma swap can be replicated by static portfolio of options with a continuum of strike prices where each option is weighted by *1/K*. The following graph displays the amount of options required to replicate the gamma and variance swaps when using the Derman et al.'s discrete hedging approximation[2].

[2] See "Volatility Investing Handbook: Variance Swaps and Beyond", September 2005 for a description of the method

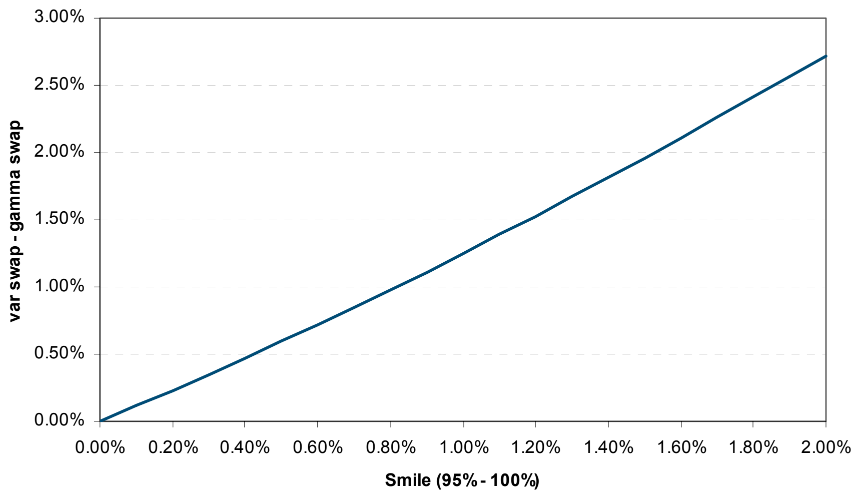
**Variance swaps and gamma swaps' replication weights using vanilla options**




Source – BNP Paribas

The gamma swap's replication therefore requires less expensive OTM option when compared with that of variance swaps. It further implies that both products have a different exposure to the smile and that by trading one against the other, one can get an exposure to the smile. The following graph shows the difference between variance and gamma swaps' strike prices for different levels of smile. Here we define the smile as the 95%-100% implied volatility spread and we keep the same average level of volatility. Therefore by increasing the smile, we steepen the volatility curve.

**Variance swap/gamma swap strike prices' spread as a function of the smile**




Source – BNP Paribas

As one can see, the relationship between the variance/gamma swap spread and the smile is increasing and almost linear.

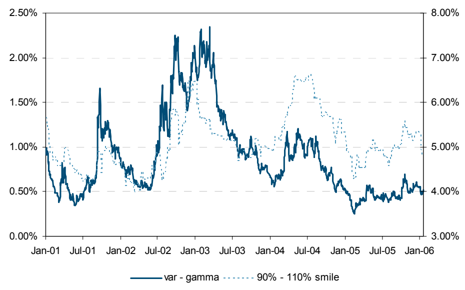
## How to make you smile – From theory to practice

The previous graph could be somewhat misleading. If one wants to buy the smile, he would simply need to buy a variance swap and sell a gamma swap. In practice, however, the situation is – as usual – different. While the

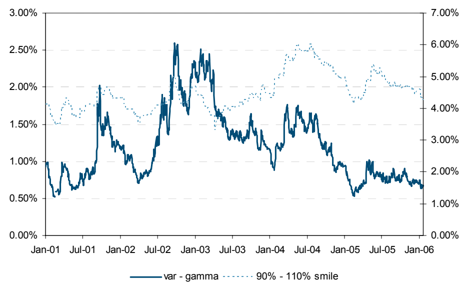
## A strong but not stationary correlation

**Variance swaps/gamma swaps spread and the 90%-110% smile (1-year)**

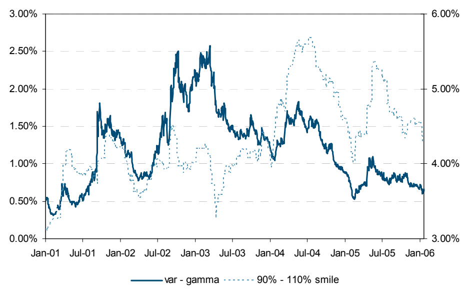
**FTSE 100**



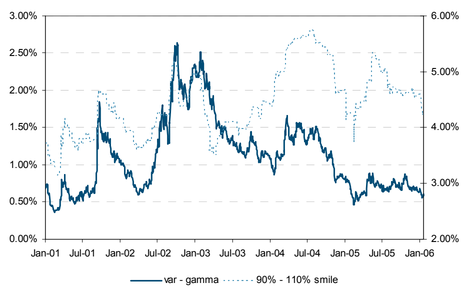

**€-STOXX 50**




**DAX 30**




**CAC 40**




Source – BNP Paribas

variance/gamma swap spread payoff is indeed a function of the smile, the smile is not the only factor affecting its payoff:

- Spot price: while the variance swap's payoff is invariant with the underlying asset price, the gamma swap is slightly delta-positive as its payoff is a function of the spot price.

- Smile structure: The sensitivity of the variance/gamma swap spread to the smile is almost linear in theory. However, as shown in the previous section, the smile is not a linear function of strike prices. The smile structure itself - and in particular its wings - could thus affect the sensitivity of the variance/gamma swap spread to the smile.

In order to study the relationship between the variance/gamma swap spread and the smile, we have simulated 1-year variance and gamma swaps' strike prices using the Derman et al.'s methodology from January 2001 to January 2006 for the major European indices. When needed, the dividend yield is calculated by taking the historical dividend yield and the bond yield is approximated by the 1-year swap rate in the relevant currency.

## The smile & the variance/gamma swap strike price spread

The following graphs display the 90%-110% smile as well as the variance/gamma swap strike price spread for the €-Stoxx 50, FTSE 100, CAC 40 and DAX 30 indices.

The two series are as expected strongly and positively correlated but it is worth bearing in mind that the correlation is not stationary. The full sample correlation ranges between 13.60% for the DAX 30 to 47.50% for the FTSE 100 but is much higher if we consider sub samples. This is explained by a de-correlation between the smile and the variance gamma spread between April 2003 and January 2004, i.e. during the strong equity market recovery. For all indices, the smile started to bounce back from very low level while the variance-gamma swap spread declined as gamma swaps prices were pricing a rise in the equity market.

**Correlation between 90%110% smile and variance/gamma strike price spread (1-year)**

| | €-Stoxx 50 | FTSE 100 | DAX 30 | CAC 40 |
|---|---|---|---|---|
| **Jan. 01 - Jan. 05** | 15.89% | 37.48% | 14.91% | 34.72% |
| **Jan. 01 - Sep. 03** | 55.90% | 78.90% | 53.39% | 79.47% |
| **Oct. 03 - Jan. 05** | 72.57% | 78.52% | 55.39% | 69.01% |

Source – BNP Paribas

## To delta-hedge or not to delta hedge? That is the question

In theory, one should delta-hedge its gamma swap in order to immunise against changes in the underlying share price. As the gamma swap's payoff should be slightly delta positive, investors/traders should take a negative position in the underlying asset or its futures.

The question is however less trivial in practice. First, the index's variance and its performance are strongly negatively correlated. When markets decline, volatility usually rises. Variance and Gamma swaps' realised payoffs are thus in reality negatively correlated to the underlying asset's performance. Therefore, since delta-hedging the gamma swap would imply taking a negative position in the underlying asset, it would increase the negative sensitivity of the gamma swap to the underlying asset's performance.

**Correlation between 1-year index's performance and 1-year variance and gamma swaps' realised payoffs**

| | €-Stoxx 50 | FTSE 100 | DAX 30 | CAC 40 |
|---|---|---|---|---|
| variance swap | -79.29% | -80.80% | -70.23% | -78.39% |
| gamma swap | -71.44% | -76.06% | -56.11% | -70.29% |

Source – BNP Paribas

What about the strike prices? Variance and gamma swaps' strike prices are defined as a portfolio of option with a continuum of strike prices. These options are weighted by $(1/K)^2$ and $1/K$ in the case of the variance and gamma swaps respectively. Since options prices are usually determined by arbitrage models, they should be independent of investors' expectations about the underlying asset's expected returns.

**Correlation between 1-year index's performance and 1-year variance and gamma swaps' strike prices lagged 1 year**

| | €-Stoxx 50 | FTSE 100 | DAX 30 | CAC 40 |
|---|---|---|---|---|
| variance swap | 15.59% | 1.00% | 35.79% | 16.46% |
| gamma swap | 11.54% | -3.80% | 32.86% | 11.54% |

Source – BNP Paribas

But, in fact, what matters is not the delta of each strategy but the delta of the variance/gamma spread. There exists several ways to delta-hedge the gamma swap. If we neglect interest rate, dividend, repo rate and if we assume a constant volatility, one can show that in order to get a delta-neutral gamma swap, one needs to hold:

$\frac{\sigma^2}{S_0}\frac{T-t}{T}$ shares at time t.

The P&L of such a delta-hedge strategy is at time T equal to:

$$\frac{\sigma^2}{S_0}\sum_{i=0}^{T-1}\left[\frac{T-t}{T}\left(S_{i+1}-S_i\right)\right]=\frac{\sigma^2\left[\bar{S}-S_0\right]}{S_0}$$

The following table shows the correlation between the 1-year index's performance and:

- The variance/gamma's payoff i.e.:

$$\sum_{i=1}^{252}\left[\left(\ln\left(S_i/S_{i-1}\right)\right)^2\left(1-\frac{S_i}{S_0}\right)\right]$$

- The variance/gamma's strike price spread: $K_{Var}$-$K_{Gamma}$

- The P&L, i.e. the difference between the payoff and the strike price spread

- The delta-hedged P&L, i.e. the P&L plus the aforementioned delta-hedging strategy

**Correlation between 1-year index's performance and variance/gamma spreads**

| | €-Stoxx 50 | FTSE 100 | DAX 30 | CAC 40 |
|---|---|---|---|---|
| var-gamma's payoffs | -89.25% | -88.42% | -87.48% | -88.73% |
| var-gamma's strike prices | 42.00% | 33.89% | 52.71% | 44.90% |
| P&L (payoffs-strike prices) | -89.31% | -85.84% | -88.76% | -89.01% |
| P&L + delta-hedge | -71.88% | -62.87% | -66.52% | -68.73% |

Source – BNP Paribas

Delta-hedging the position allows for reducing the correlation of the P&L with the performance of the index by 20% an average. However, as the correlation remains very high and negative, the benefit of delta-hedging may be overweighted by its operational and financial costs.

# A simple smile trading strategy

While a pure smile trade may be difficult to set up, trading variance swaps against gamma swaps can be optimised with respect to the smile level. We therefore suggest a straightforward trading strategy to profit from very cheap or dear smiles. If the 90-110% smile is 1.5-standard deviation above (below) its historical average, we recommend selling (buying) a variance swap and buying (selling) a gamma swap. This fairly simple "standard-deviation" signal has been profitably used in cash pairs trading for years actually[3].

We perform this strategy on four different European indices, namely the €-Stoxx 50, the FTSE 100, the CAC 40 and the DAX 30 using 1-year variance and gamma swaps. Results from the backtest are reported in the following table, assuming a 50bps bid-ask on each swap.

First, a systematic long-short strategy would have poorly performed. Buying a variance swap or a gamma swap and selling the other would have resulting in a performance on average either negative or barely positive.

Applying the "1.5 standard-deviation" screen on the smile strongly improves the performance of the strategy. For the FTSE 100 for example, being long a 1-year variance swap and short a 1-year gamma swap would have generated 1.25% in variance terms. Both strategies have performed positively in at least 75% of the case whatever the considered European index.

The number of potential trades lies between 2 to 12% of the days in our sample. This could be increased by relaxing the filter, i.e. by considering for example cases where the 90-110% smile is one standard-deviation away from its mean instead of 1.5. This would yield a potential trade in around 18% of the cases or one every six trading days but this would be accompanied by an increase of the potential loss, without necessarily a subsequent rise in the potential performance.

**Statistics on the smile trading strategy January 2001 – January 2006**

| | | €-Stoxx 50 | FTSE 100 | CAC 40 | DAX 30 |
|---|---|---|---|---|---|
| ***Long variance short gamma*** | **average** | 1.21% | 1.25% | 0.43% | 0.96% |
| | **minimum** | -2.29% | 0.56% | -1.93% | -2.90% |
| | **maximum** | 5.05% | 1.80% | 1.13% | 1.88% |
| | **percentage of positive performance** | 76% | 100% | 86% | 87% |
| | **percentage of occurrence** | 4% | 2% | 5% | 6% |
| | **systematic long-short average** | 0.08% | -0.26% | -0.02% | 0.35% |
| ***Short variance long gamma*** | **average** | 0.52% | 0.32% | 0.46% | 0.60% |
| | **minimum** | 0.31% | 0.13% | 0.29% | 0.41% |
| | **maximum** | 0.74% | 0.95% | 0.67% | 0.84% |
| | **percentage of positive performance** | 100% | 100% | 100% | 100% |
| | **percentage of occurrence** | 12% | 12% | 14% | 12% |
| | **systematic long-short average** | -0.59% | -0.17% | -0.48% | -0.89% |

Source – BNP Paribas

[3] See for example « Pairs trading: performance of a relative value arbitrage rule », by Gatev, Goetzmann and Rouwenhorst, forthcoming in the Review of Financial Studies.

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