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Variance Swaps: An Introduction

VARIANCE SWAP BASICS

A **variance swap** is an over-the-counter derivative contract in which two parties agree to buy or sell the realized volatility of an index or single stock on a future date—the **swap-expiration date**—for a pre-determined price, the **swap-strike**. The payoff for an investor who buys variance using a swap is equal to the difference between the realized and strike variance, multiplied by the notional amount of the swap: [$\$ \text{ Notional} * (\text{Realized Volatility}^2 - \text{Strike Level}^2)$]. An investor who sells variance would receive the opposite payoff: [$\$ \text{ Notional} * (\text{Strike Level}^2 - \text{Realized Volatility}^2)$]. **Variance swap vega** is the dollar-change in swap value for each percentage-point movement in volatility. The vega of the swap equals twice the volatility times the notional amount of the swap: [$2 * \text{Volatility} * \$ \text{ Notional}$]. Variance swaps have been actively traded over-the-counter for more than 8 years, and more recently (since 2004) have begun to trade on the CBOE.¹

Portfolio managers can enter into variance swaps easily. The investor specifies which underlying index to use and the duration of the swap contract. He then must determine whether he wants to buy variance (so that he will profit from any *increase* in the volatility of the index above the swap strike) or sell it (so that he will profit from any *decrease* in volatility below the swap strike). Finally, the investor must decide the type of volatility exposure. Typically, investors choose “vega exposure”—a \$100,000 vega exposure means that the swap value will change by \$100,000 for each percentage point change in the volatility of the underlying index. Once the investor has an ISDA master agreement in place, he can obtain a quote and trade.

EXAMPLE VARIANCE SWAP AND PAYOFF CALCULATION

Realized volatility is used to determine the swap payoff at maturity. This measure is calculated from daily index closing prices in a straightforward and standardized way.²

The payoff calculation from an example **30-day short S&P 500 variance swap** entered into on Feb 23, 2005 is shown below. The swap **valuation date** was April 7, 2005, the **notional swap vega amount** was \$100,000, and the swap **strike price** (ask) is 11. The swap settlement value was 9.8701, which was below the swap strike of 11; the contract was closed at a gain of \$107,183. Essentially, as volatility decreased by roughly 1 volatility point, the investor gained \$107,183.

Fig. 1: Swap Settlement for 30-Day S&P 500 Variance Swap

Day Trade	Date	Index	Log Return	Log Return ²
1	02/24/05	1,190.8	—	—
2	02/25/05	1,200.2	0.7863%	0.0062%
3	02/28/05	1,211.4	0.9264%	0.0086%
4	03/01/05	1,203.6	-0.6435%	0.0041%
5	03/01/05	1,210.4	0.5642%	0.0032%
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
23	03/29/05	1,165.4	-0.7625%	0.0058%
24	03/30/05	1,181.4	1.3679%	0.0187%
25	03/31/05	1,180.6	-0.0694%	0.0000%
26	04/01/05	1,172.9	-0.6518%	0.0042%
27	04/04/05	1,176.1	0.2725%	0.0007%
28	04/05/05	1,181.4	0.4471%	0.0020%
29	04/06/05	1,184.1	0.2266%	0.0005%
Valuation	04/07/05	1,191.1	0.5953%	0.0035%

Realized Vol Calculation	
Sum of Log Returns Squared	0.1160%
Divided by number of returns (30)	0.0039%
Square Root	0.6218%
Multiplied by square root of 252	9.8701%
Swap settlement value	9.8701

Swap Payoff	
Strike	11
Vega	\$ 100,000
Notional	\$ 4,545
Payoff	\$ 107,183

Source: Bear Stearns Equity Derivatives Strategy, Bloomberg.

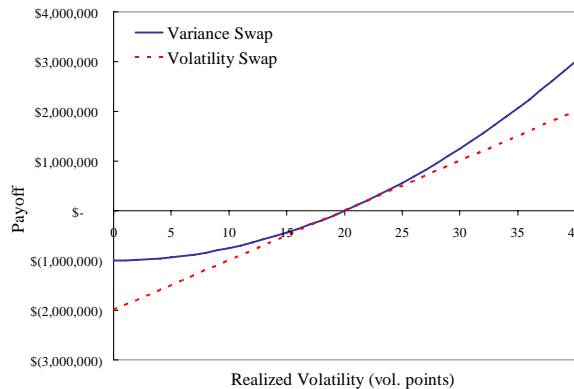
¹ 3-month variance futures now trade on the CBOE. The first documented variance swap traded on the FTSE Index in 1993.

² To calculate this, one first takes the index’s daily log-returns; then the sum of the squares of the returns is divided by the number of daily-returns measured. This volatility number is multiplied by the square root of 252 to annualize the measure, and by 100 to return a number in volatility points as opposed to a decimal number. This resulting number is the realized volatility over the life of the swap, and is the **swap settlement value**.

Variance Swaps: An Introduction

The payoff of a volatility swap is directly proportional to realized volatility; the profitability of a variance swap, however, has a quadratic relationship to realized volatility. These relationships are shown in Figure 2.

Fig. 2: Payoff of Long Variance and Volatility Swaps Struck at 20 Volatility Points; \$100,000 Notional Vega



Source: Bear Stearns Equity Derivatives Strategy, Bloomberg.

Since a long variance swap gains more than a simple volatility swap when volatility increases and loses less than a volatility swap when volatility decreases, variance swap levels are typically quoted above the expected level of future realized volatility (ie: above option-implied volatility). This spread between variance and volatility is called **convexity**. Examples of Bear Stearns' indicative levels for S&P 500 and Nasdaq 100 variance swaps are shown below.³ For example, on April 7th, an S&P 500 (SPX) variance swap with Dec 05 expiration was bid at 14.4 and offered at 15.4. A sample term sheet is shown as well.

Fig. 3: Example Quotes for Index Variance Swaps (left); Sample Term Sheet (right)

4/7/05	
SPX	FUTURES 1186 @ 9:46
May'05 Expiry	-- 11.80 / 12.80
Jun'05 Expiry	-- 12.90 / 13.90
Sep'05 Expiry	-- 13.80 / 14.80
Dec'05 Expiry	-- 14.40 / 15.40
Dec'06 Expiry	-- 15.90 / 16.90
4/7/05	
NDX	FUTURES 1490 @ 9:54
May'05 Expiry	-- 17.00 / 18.00
Jun'05 Expiry	-- 18.25 / 19.25
Sep'05 Expiry	-- 19.50 / 20.50
Dec'05 Expiry	-- 20.25 / 21.25
Dec'06 Expiry	-- 21.50 / 22.50

S&P 500 INDEX VARIANCE SWAP March 4, 2005	
Transaction Summary:	Buyer enters into a variance swap, under which it <ul style="list-style-type: none"> • Receives a Payoff at Maturity if the Volatility, as calculated on the Valuation Date, is greater than the Strike. • Pays a Payoff at Maturity if the Volatility, as calculated on the Valuation Date, is lower than the Strike.
General:	
Seller:	
Buyer:	Bear Stearns International, Limited ("BSIL")
Trade Date:	March 4, 2005
Valuation Date:	April 15, 2005
Payment Date:	3 Exchange Business Days following the Valuation Date
Volatility Units (USD per point):	USD 100,000
Variance Units (USD per point):	4,385,9649 (Equal to (Volatility Units/ (2*Strike))
Underlying Index:	S&P 500 Index ("SPX")
Strike:	11.4%
Payoff at Maturity:	Variance Units * [(Volatility) ² - (Strike) ²] If such amount is a positive number, then Seller shall make a payment to Buyer If such amount is a negative number, then Buyer shall make a payment equal to the absolute amount to Seller
Volatility:	$100\% \sqrt{\frac{252 \times \sum_{i=1}^n (\text{Return}(i))^2}{n}}$, where: $\text{Return}(i) = \ln \left[\frac{\text{Index}_i}{\text{Index}_{i-1}} \right]$ 252 = Annualization Factor n = Number of observations excluding the initial observation on Trade Date, but including the Valuation Date Index _i = The closing level of the Underlying Index i business days from the Trade Date except for Index ₁ which shall equal the closing level on the Trade Date and Index _n , which shall equal the special quotation of the Underlying Index on the Valuation Date.
Documentation:	As per ISDA
Currency:	USD
Exchange:	New York Stock Exchange
Market Disruption:	Postponement
Collateral:	Uprfront: Subject to portfolio margin calculation agreed upon Mark-to-Market: As per ISDA

Source: Bear Stearns Equity Derivatives Strategy, Bloomberg.

³ This historical, real-market example is to be used for illustrative purposes only.

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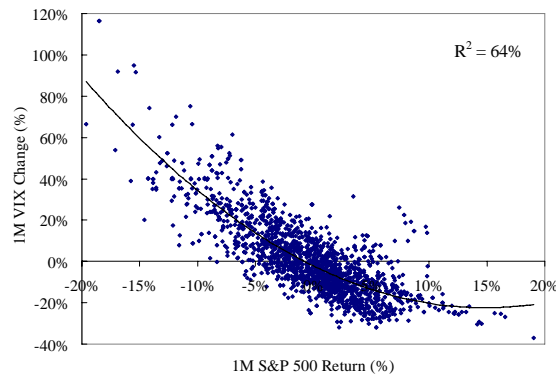
VARIANCE SWAP STRATEGIES

In this section, we list some common applications of variance swaps.

Hedging Equity Returns

Variance swaps are often used for portfolio protection. Index implied volatility (and therefore, variance) is negatively correlated to equity returns. In addition, the payoff of a long variance swap increases more rapidly than equity returns decrease. Thus, variance swaps can be used as crash-protection, insulating a portfolio from very negative equity market performance. Figure 4 shows the relationship between monthly changes in 30-day S&P 500 variance (as measured by the VIX Index) and monthly changes in the S&P 500 Index.

Fig. 4: Relationship between the S&P 500 and VIX Index; 5 Years of Monthly Returns

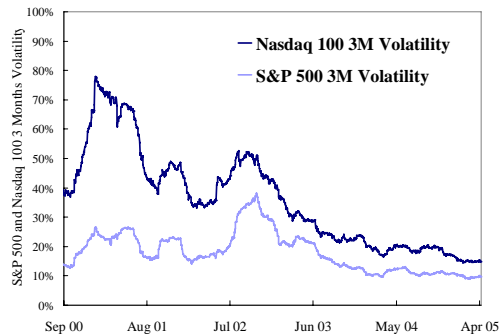


Source: Bear Stearns Equity Derivatives Strategy, Bloomberg.

Volatility Relative Value and Directional Strategies

Variance swaps can also be used in volatility-based relative value trades. For example, an investor can buy S&P 500 variance and sell a similar amount of Nasdaq 100 variance. The trade will be profitable if NDX volatility decreases relative S&P 500 volatility, or if S&P 500 volatility increases less than Nasdaq 100 volatility. This trade was very profitable in the months following the tech bubble burst in 2000. During this period, the weight of technology stocks in the Nasdaq 100 decreased significantly; this ultimately caused Nasdaq 100 Index volatility to decrease significantly as well, especially relative to S&P 500 Index volatility.

Fig. 5: 3M S&P 500 and Nasdaq 100 Index Volatility; 2000 – Present



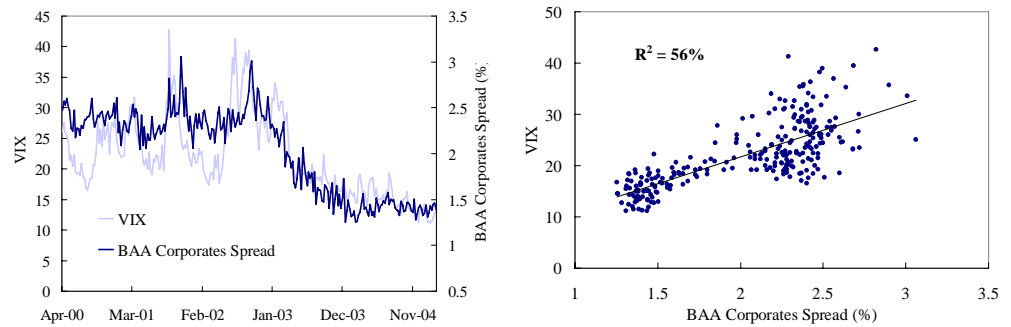
Source: Bear Stearns Equity Derivatives Strategy, Bloomberg.

Variance Swaps: An Introduction

Credit and Equity Volatility Relative Value

A company's credit risk (default risk) is related to its stock price and equity volatility. A decline in stock price usually leads to an increase in both equity volatility and credit risk (as measured, for instance, by the price of a credit default swap). The charts below illustrate the correlation between credit risk (as measured by the spread between BAA-rated US corporate 30-year bonds and the equivalent treasury rate) and volatility in the S&P 500 Index (as measured by the VIX Index) since April 2000. The correlation between these two measures was 75% during this period.

Fig. 6: Correlation between Credit Risk (Corporate Spread) and Equity Volatility (VIX)



Source: Bear Stearns Equity Derivatives Strategy, Bloomberg.

An investor could use a variance swap to hedge the credit risk of a basket of corporate bonds because of this relationship. A variance swap could also be used in a credit versus equity volatility relative value trade. Credit spreads and equity volatility have been positively correlated on the single-stock level; thus, single-stock variance swaps could be used for company-specific capital structure trades.

Selling Equity Risk

The implied volatility of the S&P 500 has traded at a persistent premium to realized volatility, as a result of supply/demand inefficiency; there has consistently been excess demand for put-protection in the S&P 500. An investor can capitalize on this market inefficiency by selling variance swaps to capture the resulting premium. An investor can create several types of long-cash, short-variance portfolios that have S&P 500 "variance bonds." An example **Notional Variance Bond** is created by lending \$100M at the risk-free rate and selling \$20,000 notional variance; an example **Vega Variance Bond** is created by lending \$100M at the risk-free rate and selling \$800,000 notional vega variance. By modifying the amount of variance sold, an investor can tune the degree of leverage for a given strategy to prevent large losses should the volatility spike. Historical returns for these strategies are shown in the table below.

Fig. 8: 15-Year Historical Returns of Sample S&P 500 Variance Bonds

	Return	Risk	Information Ratio
S&P 500 Index	9.3%	15.1%	0.62
Notional Var Bond	16.5%	9.0%	1.83
Vega Var Bond	15.7%	6.6%	2.38

Source: Bear Stearns Equity Derivatives Strategy, Bloomberg.

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