Stochastic Interest Rates: A Crucial Correlation

Because it ignores the difference the correlation effect can make to net value, a single-factor model doesn't quite get the job done

To calculate the value of a stock option, you need to model stochastic share prices. To determine what a callable bond is worth, you must model stochastic interest rates. But to evaluate the options inherent in a convertible bond—a hydra with one head fixed income and the other equity—you must treat both the share price and the interest rates as stochastic variables simultaneously.

The function for valuation of convertible bonds, OVCV, is based on a two-factor model that treats both share prices and interest rates as stochastic variables. And only by means of a two-factor model can you effectively take into account the correlation between interest rates and share prices.

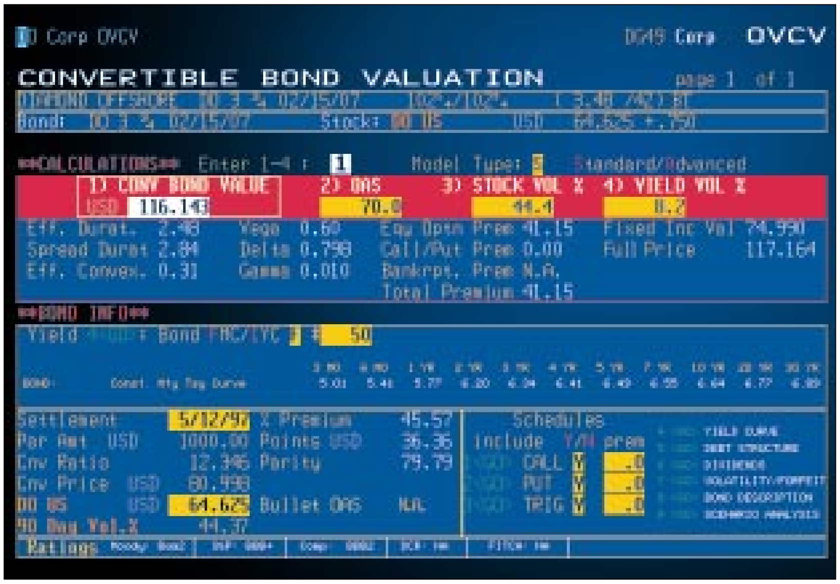

Figure 1. Type DO <Corp> OVCV <Go>. Tab in, and enter 70 in the OAS field. Press <Go> to see Diamond Offshore Drilling's convertible bond value

But because of developmental costs and the computing power needed to run a two-factor model, many convertible bond models attempt to make do with a single factor, with interest rates held constant. Indeed, most practitioners believe that although using stochastic interest rates makes a calculation slightly more accurate, the net difference is not large and so not worth the added complexity. For example, Michael Brennan and Eduardo Schwartz looked at the price difference between using constant interest rates and stochastic ones for the same convertible at various interest rates and concluded that "for a reasonable range of interest rate levels the errors from the certain interest rate model are likely to be slight, and therefore, for practical purposes

OVCV: A multitalented function

In calculating the value of a convertible bond, the OVCV function models not only stochastic interest rates but also both hard and soft calls, puts, and even dilution—à la warrants. In addition, it models dividends and credit spreads, and the advanced model even treats bankruptcy.

You can use OVCV to:

- break a convertible's value into its component parts
- calculate sensitivities on both the equity and interest-rate sides: vega, delta, gamma, duration, and convexity
- determine the implied values of share volatility or credit spread
- price dual-currency bonds, using interest-rate parity to calculate implied forward foreign exchange rates
- plot graphs that show how values change under varying scenarios

The data needed for these calculations appear on wake-up, so there's no need to search for or enter them: the yield curve, stock price, historical volatility, hard and soft calls, put schedules, and cash flows are all instantly available.

OVCV offers you a powerful two-factor model and all the data needed to run it. No slide rule required. —*E.B. & D.K.*

The correlation calculation

To calculate the correlation between interest rates and share prices, we must first understand exactly which variables we are trying to calculate the correlation of. To do that, we use the defining equations for the joint two-variable stochastic process:

$$\frac{dS}{S} = \mu_S dt + \sigma_S dZ_1 \qquad \frac{dr}{r} = \mu_r dt + \sigma_r dZ_2$$

Here σ_S and σ_r are the (constant) volatilities of the two processes and μ_S and μ_r are their drifts. Under a risk-neutral measure, $\mu_S = r$ and μ_r matches the observed yield curve. The expressions dZ_1 and dZ_2 are increments of a unit Brownian motion, and it is the correlation of $\sigma_S dZ_1$ and $\sigma_r dZ_2$ that is needed. This is exactly the same as the correlation between $d \log S$ and $d \log r$—by Ito's lemma.

We thus form the series $x_i \equiv \log S_{i+1} - \log S_i$ and $y_i \equiv \log r_{i+1} - \log r_i$ and calculate their correlation. The series x_i is just the log of the return. The sample mean and standard deviation of x_i are

$$\mu_x = \sum_{i=1}^{n} x_i \Big/ n \quad \text{and} \quad \sigma_x = \sqrt{\sum_{i=1}^{n} (x_i - \mu_x)^2 \Big/ (n-1)},$$

respectively.

The correlation between the two series is then given by

$$\rho_{xy} = \sum_{i=1}^{n} \frac{x_i - \mu_x}{\sigma_x} \frac{y_i - \mu_y}{\sigma_y} \Big/ (n-1).$$

Fortunately, you don't have to calculate these correlations yourself; CORR calculates them for you.

Type CORC 1 <Go> (if this slot is taken, use a number other than one), and enter the data as in figure 3. We have chosen USG1YL as a proxy for interest rates because, being a weighted average of bonds with maturities out to three years, it is less susceptible to Federal Reserve control. It is also one of the indexes used in standard value-at-risk calculations. Note how we use the differences in the logs in the correlation calculation by entering a *D* in the first column and an *E* in the sixth column.

Now type 2 <Go> to get the correlation matrix. Figure 4 shows that the S&P 500 has a correlation close to minus 0.5, whereas the EMC convertible has a correlation of minus 0.294. —*E.B. & D.K.*

it may be preferable to use this simpler model [constant interest rate] for valuing convertible bonds" ("Analyzing Convertible Bonds," *Journal of Financial and Quantitative Analysis* 15:4 [November 1980]).

That claim may be correct for the primary bond market, but it's not generally valid in the secondary markets. Take, for example, a busted callable convertible—a convertible whose share price is so far below its conversion price that its conversion option is almost worthless—that's paying a coupon close to par in light of the current yield curve. Such a bond will behave exactly like a normal callable bond, and to ignore stochastic interest rates in valuing such a convertible would be as inaccurate as it would be in the normal callable bond market.

But as an investor in convertible bonds, why should you care about such a bond? Because it's busted and behaves like a normal callable bond, you can leave it to the fixed-income department. After all, it isn't *really* a convertible bond now, anyway.

Fair enough, except there's more to it than that. You need to model the stochastic interest rates because share price movements and interest-rate movements are correlated and the correlation strongly affects the valuation. That point—that stochastic interest rates are important because of the correlation effect—seems to be continually overlooked by researchers and by almost all market practitioners.

To get a sense of how important the correlation effect is, consider

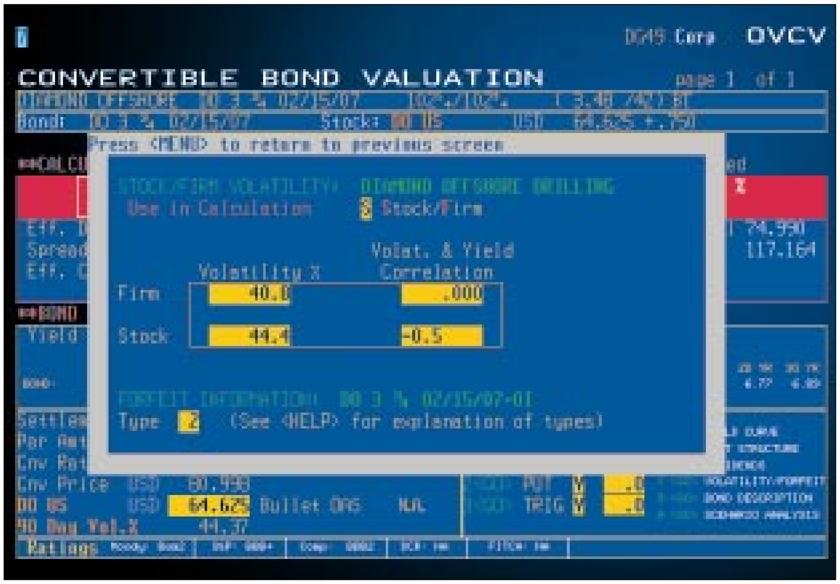

Figure 2. Type 7 <Go> from the OVCV screen. Tab down and enter –0.5 in the VOLAT. & YIELD CORRELATION field; press <Go>. Type <Menu> 99 <Go> to recalculate the bond value

what happens in a two-factor model if interest rates rise. As in the Black-Scholes equation, the risk-neutral drift of the share value—which equals the interest rate—increases. This means that future share prices are expected to be higher, which causes the option value to *increase*, making the convertible worth *more*. But everyone knows that when interest rates go up, the stock market drops. Here's the rub: higher interest rates *do* imply that the share price will drift up at a higher risk-neutral rate, but first it will drop sharply because of negative correlation with interest rates. Ignoring correlation, then, is not a smart move.

It follows, therefore, that negative correlations should lower the value of a convertible, whereas positive correlations should make it worth more. Viewed another way, a convertible's fixed-income value is an average over all interest-rate scenarios, with high values when interest rates are low and with low values when rates are high. With positive correlation, the region in which interest rates are high is more than compensated for by the higher share prices and therefore higher conversion value.

Here's how negative correlation looks in practice: Diamond Offshore Drilling issued close to half a billion dollars of 3.375 percent convertible bonds due February 15, 2007. Running OVCV with an option-adjusted spread of 70 basis points brings up a value of $116.143 (figure 1). Increasing the yield volatility from 8.2 percent changes the price by only 11 cents, to $116.251. This, of course, dovetails well with research that claims that a stochastic interest rate has little effect on a bond's value.

But let's make a slight adjustment. Type 7 <Go>, tab in to the VOLAT. & YIELD CORRELATION field, and type –0.5 <Go> <Menu> 99 <Go> to recalculate the value with a correlation of minus 0.5 (figure 2).

The price drops down to $109.618—a whopping $6.50 difference and hardly insignificant. Raising the correlation to plus 0.5 moves the value up to $122.91, for a $13 difference between a correlation of minus 0.5 and plus 0.5. On an issue of $400 million, $13 translates into $52 million.

Figure 3. Type CORC1 <Go>, and modify the correlation matrix table as shown. Type <Go> 1 <Go> to update

As one would expect, most equities have a negative correlation with interest-rate movements. The Standard & Poor's 500-stock index has a correlation of about minus 0.5, whereas Diamond Offshore Drilling has a correlation close to zero. EMC Corp., which also has a recent issue of almost half a billion dollars' worth of 3.25 percent convertibles due March 15, 2002 (EMC3.25 02 <Corp>), has a correlation of minus 0.294 (figures 3 and 4 and sidebar The Correlation Calculation).

In the EMC bond, changing the yield volatility from 8.2 percent alters the bond price by only a few cents. Changing the correlation to minus 0.294, however, decreases the value by $2.50. On an issue of $450 million, $2.50 translates into $11.25 million—which adds up to a very good reason to pay close attention to fluctuating interest rates and the correlation effect.

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Eric Berger, Ph.D., and David Klein, Ph.D., developed the Bloomberg convertible bond model

Figure 4. Type 2 <Go> from the updated CORC1 screen to see the correlation between interest rates, the Standard & Poor's 500-stock index, and EMC Corp.